

O Level A Maths Tutorial 9: Proofs in Plane Geometry

Syllabus :

- properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles *

* These are properties learnt in O-Level Mathematics

1. (a)

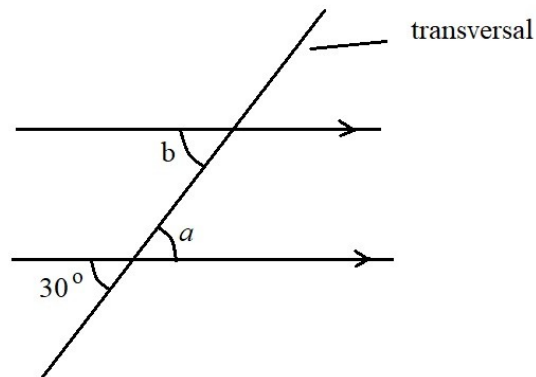


Figure 9-1

Find the values of angles a and b . State the reasons.

(b)

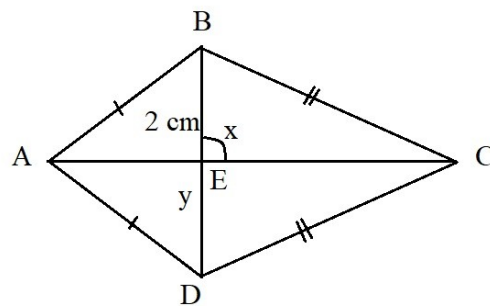


Figure 9-2

State the value of angle x . Give the reason.

What is length y ?

(c)

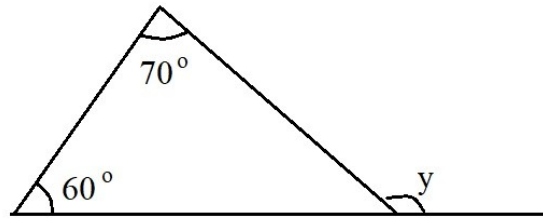


Figure 9-3

Find the value of angle y . Give the reason.

(d)

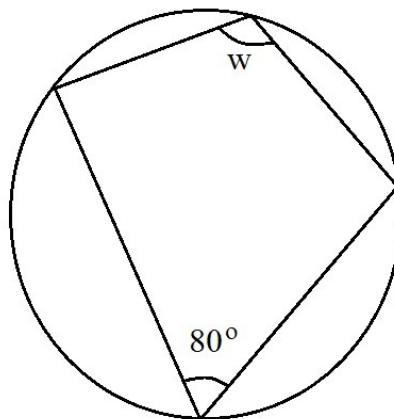


Figure 9-4

Find the value of w . State the reason.

• Use of congruent and similar triangles *

2. Two triangles are congruent if corresponding sides and angles are all equal. In the following, we only have part of the information. State which pairs are definitely congruent. If a pair are not congruent, explain why.

(i) 3 sides equal (SSS)

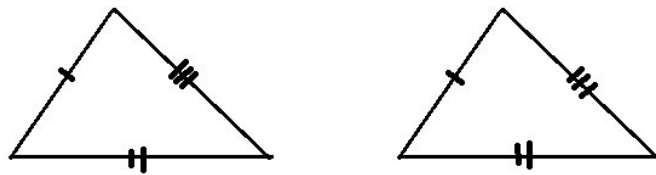


Fig. 9-5

(ii) 2 sides 1 angle (in between) equal (SAS)

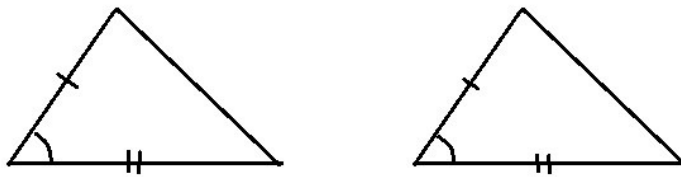


Fig. 9-6

(iii) 2 angles 1 side (in between) equal (ASA)

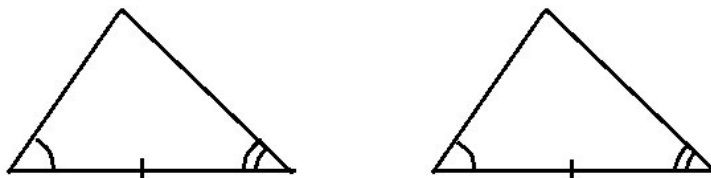


Fig. 9-7

(iv) 2 angles 1 side (not in between) equal (AAS)

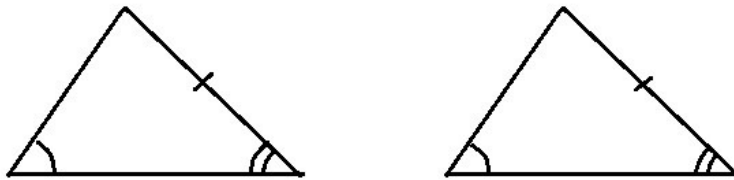


Fig. 9-8

(v) 2 sides 1 angle (not in between) equal (SSA)

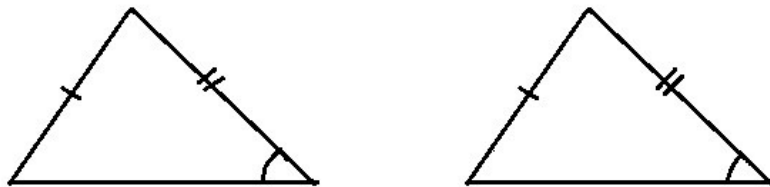


Fig. 9-9

3. Two triangles are similar if they have the same shape – all angles same – even if their sizes are different. State which of the following pairs of triangles are similar. Give the reason for each case.

(i)

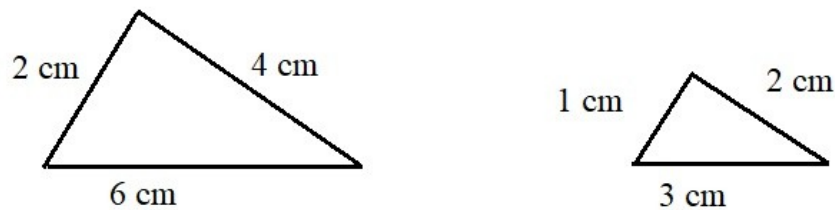


Figure 9-10

(ii)

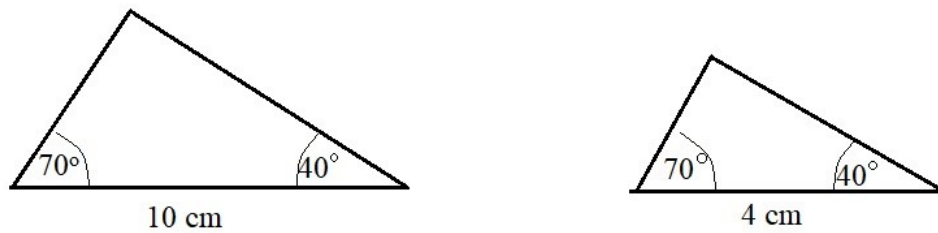


Figure 9-11

(iii)

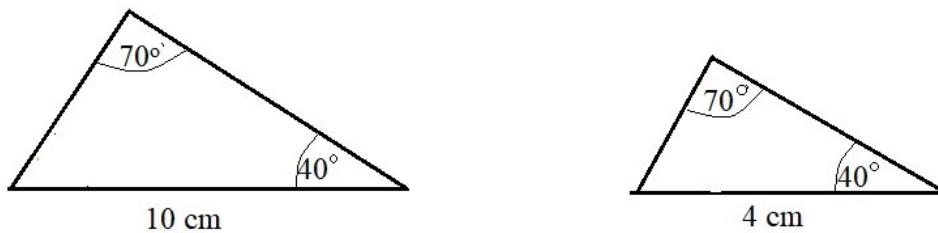


Figure 9-12

- midpoint theorem

4. “The midpoint theorem states that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.”

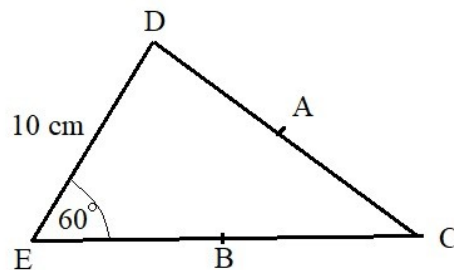


Figure 9-13

(i) What is the length of AB?

(ii) What is the angle ABC? Give the reason.

- tangent-chord theorem (alternate segment theorem)

Note: The following are examples of basic concepts lead up to an example on the alternate segment theorem. These basic concepts are covered more extensively in O level E Maths.

5. Explain why $a = 2b$.

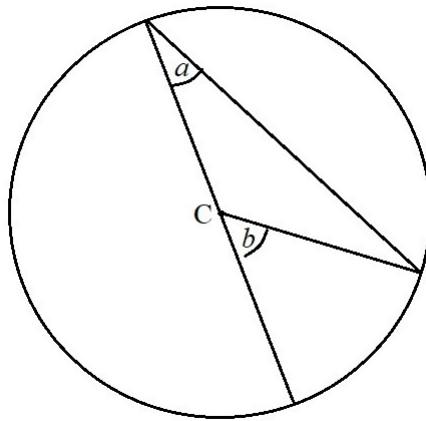


Figure 9-14

6. Show that $y = 2x$.

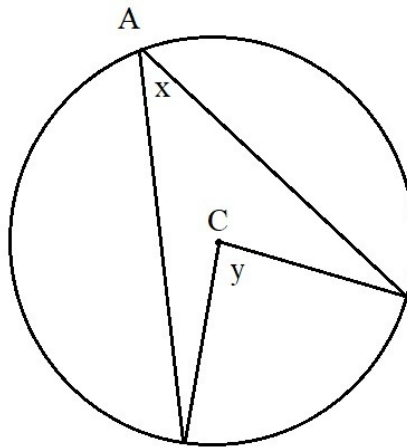


Figure 9-15

7. Show that $x = 2y$.

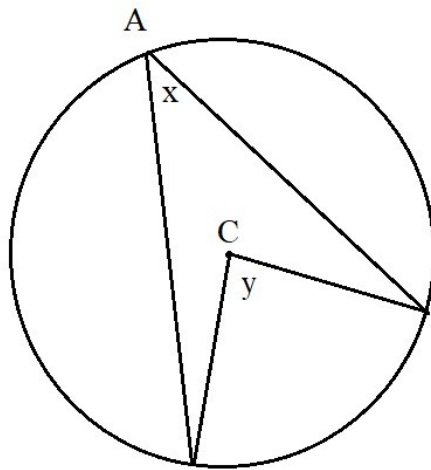


Figure 9-16

8. Show that $y = 2w$.

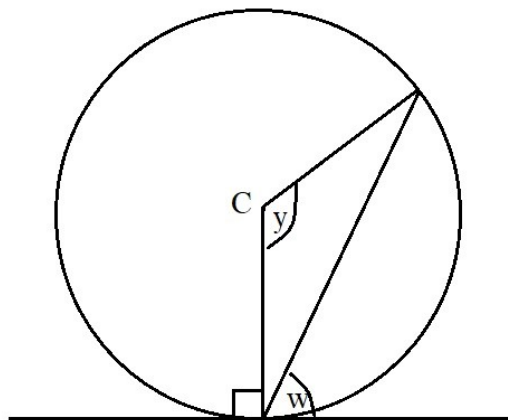


Figure 9-17

9. Show that $x = w$. (alternate segment theorem *)

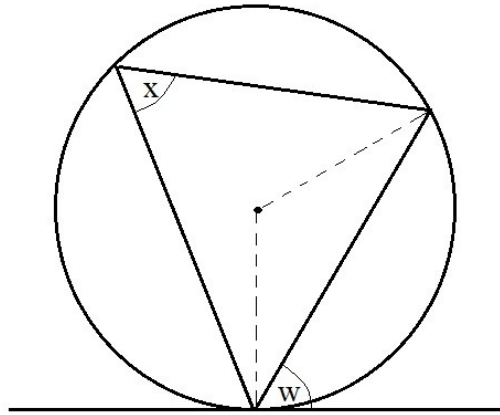


Figure 9-18

* The Alternate Segment Theorem states that the angle between a tangent and a chord at the point of contact is equal to the angle in the alternate segment formed by the chord.